Gyanveer University
Sagar (M.P.)

Ph.D Entrance Test Syllabus
Session: 2024-25
Mathematics

Part A : Research Methodology
Part B : Mathematics

Part-A
Research Methodology

Research and Types of Research:
Meaning and importance of Research, Objectives of Research, Motivation in Research. Research methods Scientific method vs Arbitrary Method, Logical Scientific Methods: Deductive, Inductive, Deductive-Inductive, pattern of Deductive, Inductive logical process, Different types of inductive logical methods, Types of research, Descriptive vs. Analytical, Applied vs. Fundamental, Quantitative vs. Qualitative, Conceptual vs. Empirical. Research Process. Criteria of good Research, Writing a survey and identifying the problem. Planning and writing a research proposal**:** Research projects, Major funding agencies, Concepts in ethics, Intellectual Property Rights, Plagiarism, Fraud and misconduct in science, Unscientific practices in thesis work.

Collection and analysis of data:
Designing experiments for the purpose of testing research hypotheses, and evaluating the results of those experiments, Analysis of data, Descriptive statistics, Inferential Statistics, Some common statistical tests and inferences. Tables, Illustrations, Graphs and visualization.

Scientific writing & Information sources:
Structure and components of Scientific Reports, types of Report, Technical Reports and Thesis Significance, Different steps in the preparation, Layout, structure and Language of typical reports, Bibliography, Referencing and foot notes, Importance of Effective Communication. Types of publications, Indexing and abstracting services, Online library, Search engines, Citation indexes, Citations analysis, Online searching methods, how to cite and list correctly, Common documentation styles, Citation of sources in the text, Reference management software's.

Preparation of thesis and research papers:

Structure of a thesis/research article/ review article, including title, introduction, literature review, methods and materials, referencing. Editing and proofreading, Use of abbreviations.

Part-B
Mathematics

Linear Algebra

Vector space, Subspace, linear dependence, Basis and dimension, Linear transformation, Kernel of a transformation, Rank and nullity of transformation, Vector space L(U,V), Matrix associated with a linear transformation and vice-versa, rank and determinant of matrices, Eigen-values and Eigenvectors, Diagonalization of Matrices, Cayley-Hamilton Theorem, Linear functional, Dual basis, Computing of a dual basis, Dual vector spaces, Annihilator, Second dual space, Invertible Linear transformation, Invariant Subspaces, Canonical form, Jordan Form, Forms on vector spaces, Bilinear Functionals, Symmetric Bilinear Forms, Skew Symmetric Bilinear Forms, Rank of Bilinear Forms, Quadratic Forms, Classification ofReal Quadratic forms. Dual transformations, Minimal Polynomial, Associative Rings and Algebras

Set Theory

Countable and uncountable sets, Equivalence relation and partition, Fundamental theorem of equivalence relation, functions, their restrictions and extensions, Equivalent sets, Infinite sets, the continuum, Cardinals, Cardinal arithmetic, Inequalities and cardinal numbers, Cantor' s theorem, Schroeder Bernstein theorem, Continuum hypothesis.

Ordinal numbers and their inequalities, Addition and multiplication of ordinals, Distributive laws, Structure of ordinal numbers, Axiom of choice, Zom*s lemma and well ordering principle, Fundamental theorem of arithmetic, divisibility in Z, congruences.

Calculus

Limit o f a function, Continuous functions, Derivative o f a function, Applications of derivatives, R olle's Theorem, mean value theorem of differential calculus and their applications. Functions of several variables, level curves and level surface, partial derivatives and directional derivatives, linearization and differentials, extrema of two variables function, curl and divergence o f a vector field. Double integral and triple integral, line integral, Green's theorem, surface area and Stokes theorem.

Group Theory

Groups, Subgroups, Normal subgroups, Cyclic groups, Generators, Lagrange's theorem, Isomorphism of groups, Quotient groups, Fundamental theorems of homomorphism's in groups, Permutation, Permutation groups, Cayley's theorem, Relation of conjugacy, Conjugate classes o f a group, Class equation in a finite group and related results, Partition o f a positive integer, Conjugate classes in Sn, Sylow 's theorems, External and internal direct products and related results. Nilpotent groups, relation between solvable and nilpotent groups, Composition series o f a group, Zassenhaus theorem, Schreier refinement theorem, Jordan-Holder theorem for finite groups.

Advanced Ring Theory

Examples and fundamental properties of rings. Direct and discrete direct sum of rings. Ideals generated by subsets and their characterization in terms of elements of the ring under different conditions. Sum and direct sums of ideals, Ideals products and nilpotent ideals. Minimal ideals, Complete matrix ring, Idea is in complete matrix ring. Residue class ring. Necessary and sufficient conditions for an ideal to be a prime and semiprime ideal, Prime radical o f a ring, Prime rings and its characterization in terms of prime ideals, Primness of complete matrix rings. D.C.C. for ideals and the prime radical, Jacobson radical and simple properties, Relationship between Jacobson radical and prime radical o f a ring.

Field Theory

Field Extensions, Algebraic extensions, Splitting fields, Normal extensions, Multiple roots, Finite fields and separable extensions, Automorphism groups ad fixed fields, Fundamental Theorems of Galois Theory, Fundamental Theorem of Algebra. Einstein's criterion, Gauss theorem.

Real Analysis

Limit o f a function, sequential approach, Cauchy's criteria for finite limits, Continuous and discontinuous functions and their properties, Uniform continuous functions, Increasing and decreasing functions, Darboux's theorem, Taylor's theorem, Maculaurin's theorem, Taylor's and Maculaurin's infinite series with applications, Functions ofbounded variations and their properties, Variation of functions, Jordon theorem. Limit points o f a sequence, Types of limits, Convergent and divergent sequences and related results, Cauchy's general principle of convergence. Algebra of sequences and related results with applications, Monotonic increasing and decreasing sequences, Infinite series, Convergence of infinite series, Positive term series, various type of tests for convergence of series, Alternating series, Leibnitz test, absolute and conditional convergence, Series of arbitrary terms, A bel's and Dirichlefs tests. Riemann integral, Darboux' s theorem.

Complex Analysis

Functions of a Complex variable, Limits, Continuity and Derivatives of functions, Cauchy Riemann equations and related results, Analytic functions, Harmonic functions, Harmonic conjugate, complex valued functions, Complex line integrals and contour integrals, power series, Taylor's series: Laurent's series, fundamental theorem of line integrals (or contour integration), Cauchy's theorem, Cauchy-Goursat theorem, symmetric, Cauchy's theorem for a disk, Cauchy's integral theorem, index o f a closed curve, advanced versions of Cauchy integral formula with applications, Cauchy's estimate, Morera's theorem, Convergence of sequences and series of complex valued functions, Weierstrass' M-test, power series as an analytic function, root and ratio test, uniqueness theorem for power series, zeros of analytic functions, identity theorem and related results, maximum/minimum modulus principles and theorems, Schwarz' lemma and its consequences, isolated and non-isolated singularities, removable singularities, poles.

O .D .E. and P.D .E.

Linear differential equations of order n, complementary functions and particular functions, Cauchy-Euler's and Legendre linear equations, series solution of differential equations with variable coefficients, Initial value problems and boundary value problems, linear dependence and independence of solutions, equations with constant as well as variable coefficients, Wronskian, variation of parameter, method of undetermined coefficients, reduction of the order of an equation, method of Laplace's transform, Lipschitz's condition and GronWall' s inequality, Picards theorems, dependence of solution on initial conditions and on the function, Continuation of solutions, Non-local existence of solutions Systems as vector equations, existence and uniqueness of solutions, Strum-Liouvilles system, Green's function and its applications to boundary value problems, some oscillation theorems such as Strum theorem, Strum comparison theorem and related results, System of first order matrix equation, fundamental matrix, Non-homogeneous linear system, Linear system's with constant as well as periodic coefficients.

Differential Geometry

Curves, curvature and torsion, Seret-Frenet Formulae, fundamental theorem of curves, surfaces, first and second fundamental forms, Gaussian curvature, curves on surfaces, geodesic, normal curvatures, principal curvature, Charts, Atlases, Manifolds, Differentiable structure on a manifold, Smooth maps, Tangent vectors and Iangent space, Vector fields, Lie product of Vector fields, Jacobian of a smooth map, Integral curves on a manifold, Cotangent spaces, pullback of l-forms, Tensor fields, Differential forms, Exterior product and derivative, Connexion, parallelism, Geodesic, Covariant differentiation, Torsion, Curvature, Structure equation of Cartan.